# Section 7 Frictional Forces: The Mu of the Shoe

**Florida Next Generation Sunshine State Standards:**
***Additional Benchmarks met in Section 7***

**SC.912.P.12.3** Interpret and apply Newton's three laws of motion.

**MA.912.S.3.2** Collect, organize, and analyze data sets, determine the best format for the data and present visual summaries from the following: bar graphs, line graphs, stem and leaf plots, circle graphs, histograms, box and whisker plots, scatter plots, cumulative frequency (ogive) graphs.

## What Do You See?

## What Do You Think?

A shoe store may sell as many as 100 different kinds of sport shoes.

- **Why do some sports require special shoes?**
- **Why would different features of a shoe be useful for different sports?**

Record your ideas about these questions in your *Active Physical Science* log. Be prepared to discuss your responses with your small group and the class.

## Investigate

In this *Investigate*, you will examine how difficult it is to pull a shoe across a surface.

1. Take an athletic shoe. Use a spring scale to measure the weight of the shoe, in newtons.

   a) Record a description of the shoe (such as its brand) and the shoe's weight, in your log.

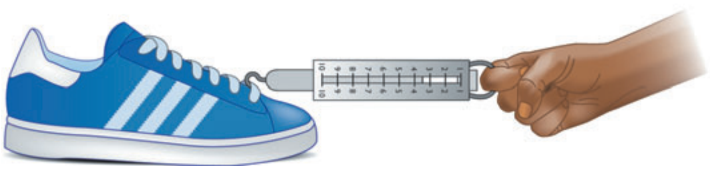
   b) List some things (which scientists call "variables") that may affect the force required to pull the shoe.

2. Design an experiment that would allow you to determine how one of the variables you listed affects the force required to pull the shoe. Include in your design:
   - What you will be able to conclude as a result of your experiment.
   - What data you will record.
   - What tools you will use to measure your data.
   - How you will analyze your data.

   a) Record your procedure in your log.

Your teacher may ask you to continue with your experimental design or, because of equipment or safety concerns, ask you to proceed with the experiment as described below.

3. Your teacher may ask you to use blocks of wood instead of shoes for greater precision in your results. However, wooden shoes are not recommended for sports. Place the shoe on a horizontal surface (either rough or smooth) designated by your teacher. Attach the spring scale to the shoe as shown below (low down at the toe or heel or hooked onto the front lace loop) so that the spring scale can be used to slide the shoe across the surface while, at the same time, the scale can be read. Be sure to keep the spring scale parallel to the surface. Alternatively, you might use a force sensor attached to a computer. The computer then displays the force reading.



   a) Record a description of the surface in your log.

   b) Measure and record the amount of force, in newtons, needed to keep the shoe sliding on the surface at a slow, constant speed. Be careful to pull horizontally so that you do not tend to lift the shoe or pull downward on the shoe. Do not measure the force needed to start the shoe moving. Measure the force needed to keep it sliding at a slow, constant speed. How will you determine if the shoe is moving with a constant speed?

When the shoe is being pulled across the surface, there are four forces acting on it. The first is the horizontal force you apply and is measured by the spring scale. But since the shoe moves at a slow, constant speed, it is not accelerating horizontally. There must be a second force on the shoe of equal strength and in the opposite direction to the force you apply. This second horizontal force is the force due to *friction* between the shoe and the surface. The third force is the downward force of gravity on the shoe, which is equal to the weight of the shoe. But since the shoe is not accelerating downward, there must be a fourth force on the shoe. This is the force of the table on the shoe; this force is equal in strength and in the opposite direction to the shoe's weight. Since this force is directed perpendicularly to the surface, it is often called the *normal force*, since the word "normal" sometimes means "perpendicular to."

The *coefficient of sliding friction*, symbolized by the Greek letter $\mu$ (mu), is calculated using the following equation:

$$\mu = \frac{\text{force of friction}}{\text{perpendicular force exerted by the surface on the object}}$$

Example:

Brand X athletic shoe has a weight of 5.0 N. If 1.5 N of applied horizontal force is required to cause the shoe to slide with constant speed on a smooth concrete floor, what is the coefficient of sliding friction?

The force of friction is equal to the applied horizontal force because there was no acceleration. The perpendicular force exerted by the surface on the shoe must be equal to the force exerted by the shoe on the surface (which is the weight of the shoe). In this case, that would be the same as the weight of the shoe.

$$\mu \text{ on concrete} = \frac{1.5\ \cancel{\text{N}}}{5.0\ \cancel{\text{N}}} = 0.30$$

c) Use the data you have gathered to calculate $\mu$, the coefficient of sliding friction for this particular kind of shoe on the particular kind of surface used. Show your calculations in your log.

4. Add something to the shoe to approximately double its weight. Pull the shoe with a spring scale.

a) Record the force to pull the heavier shoe.

b) Calculate $\mu$ for the heavier shoe, showing your work in your log.

c) Taking into account possible errors of measurement, does the weight of the shoe seem to affect $\mu$? Use data to answer the question in your log.

d) How do you think the weight of an athlete wearing the shoe would affect $\mu$? Why?

5. Place the shoe on the second surface designated by your teacher. Repeat the procedure.

a) Make a sketch (free-body diagram) to show the forces acting on the shoe.

b) Calculate $\mu$ for this new surface and the shoe.

c) How does the value of $\mu$ for this surface compare to the value of $\mu$ for the first surface used? Suggest reasons for any difference in $\mu$.

d) Would it make any difference if you used the empty shoe or the weighted shoe to calculate $\mu$ in this step? Explain your answer.

## Physics Talk

### FRICTION

#### Analyzing the Forces Acting on the Shoe

In this *Investigate*, you pulled the shoe at a constant velocity. Newton's second law informs you that motion with a constant velocity happens only when there is no net force on the shoe. So all the forces on the shoe must add up to zero.

You applied a horizontal pulling force and measured the value of the force with the spring scale. The shoe moved at a slow, constant speed. It was not accelerating horizontally. Therefore, there must be a second force on the shoe of equal strength and in the opposite direction to the force you applied. This second force was the force due to **friction** between the shoe and the surface. The pulling force you applied was equal to the frictional force, and since the two forces were in opposite directions, the net or

**Physics Words**

**friction:** a force that resists relative motion between two bodies in contact.

total force due to them was zero. Note that you actually measured the pulling force but used its value as the value for the frictional force. This is perfectly fine, since the two forces are equal in strength.

In the *Investigate*, the shoe did not move in the vertical direction. Newton's second law informs you that the vertical forces on the shoe must add up to zero. The downward force of gravity on the shoe (weight) must be equal to the upward force applied to the shoe by the surface. Since this force is directed perpendicularly to the surface, it is often called the **normal force**, since the word "normal" sometimes means "perpendicular to." This force is equal in strength and in the opposite direction to the shoe's weight. Note that you measured the weight of the shoe, but used its value as the value for the normal force. Again, this is perfectly fine, since the two forces are equal in strength.

A free-body diagram can help you see the relationships among the four forces when the shoe moves with a constant speed.


### Coefficient of Sliding Friction, $\mu$

The **coefficient of sliding friction**, symbolized by $\mu$, is defined as the ratio of two forces:

$$\mu = \frac{\text{force of friction}}{\text{perpendicular force exerted by the surface on the object (normal force)}} = \frac{F_f}{F_N}$$

The force of friction is equal to the force required to slide the object on the surface with a constant speed.

**Physics Words**

**normal force:** the force acting perpendicularly or at right angles to a surface.

**coefficient of sliding friction:** a dimensionless quantity symbolized by the Greek letter $\mu$; its value depends on the properties of the two surfaces in contact and is used to calculate the force of friction.

Note the following about the coefficient of sliding friction:

- $\mu$ does not have any units because it is a force divided by a force; it has no unit of measurement.
- $\mu$ usually is expressed in decimal form, such as 0.85 for rubber on dry concrete (0.60 on wet concrete).
- $\mu$ is valid only for the pair of surfaces in contact when the value is measured; any significant change in either of the surfaces (such as the kind of material, surface texture, moisture, or lubrication on a surface, etc.) may cause the value of $\mu$ to change.
- The situation in this section was chosen deliberately so that the "perpendicular force exerted by the surface on the object" was exactly equal to the weight. If the surface were tilted, or if the pulling force were angled upward or downward, then the force exerted by the surface on the object would be more difficult to determine.

### The Greek Alphabet

There are not enough letters in the English alphabet to provide the number of symbols needed in physics, so letters from another alphabet, the Greek alphabet, are also used as symbols. The Greek alphabet has been used for centuries to write the Greek language. It is one of the oldest alphabets in use today.

The letters of the Greek alphabet are often used in physics and mathematics. The letter $\mu$ (pronounced like "mew" and rhymes with "you") traditionally is used in physics as the symbol for the "coefficient of sliding friction." There are other Greek letters you may use in *Active Physical Science* or other physics courses that are shown below. For example, you have already used the Greek letter "$\Delta$" to represent "a change in."

## Checking Up

1. Why can you say that the force of friction is equal to the force reading on the spring scale when pulling an athletic shoe across a surface with a constant speed?
2. Why does the coefficient of friction have no units?
3. What determines the coefficient of friction?

Active Physics Plus

| +Math | +Depth | +Concepts | +Exploration |
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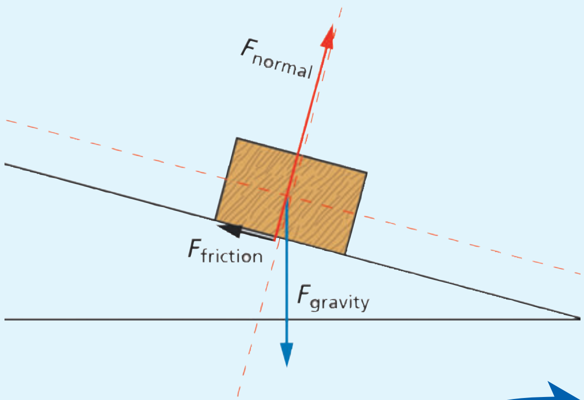
## Static Friction

As you worked on this *Investigate*, you might have noticed that it takes a larger force to get an object sliding across a surface than to keep it sliding once it has started to move. In that section, only sliding friction is discussed. When the object is not sliding, friction still acts between the surface and the object, but the force of friction now assumes the appropriate value between zero and a maximum so that the object remains at rest.

Imagine the athletic shoe at rest and you are not pulling on it. Now start to pull very gently. You are clearly applying a force to the shoe, but since the shoe is still at rest, the force of static friction must be equal and opposite to the force you are applying. Now pull on it a bit harder. If the shoe does not move, then the frictional force has also increased, so it is still equal and opposite to the force you are applying. Notice that the static frictional force can take on various values. This is unlike the sliding frictional force that always has a definite value for a given situation. If you keep increasing your pull on the shoe, the static frictional force also keeps increasing, until it reaches its maximum value given by $F_x = \mu_S F_N$. As soon as the force you are applying is greater than the maximum static frictional force possible, the shoe breaks loose and accelerates. Because this force is greater than the sliding frictional force ($\mu_S > \mu$), you have to decrease the force you exert on the shoe in order for it to slide with a constant velocity. This is something that you may have noticed.

1. A block sitting on an incline makes an angle of 30° with the horizontal. The block has a mass of 1.5 kg.
   a) Sketch the block. Draw three arrows representing the forces on the block (weight pointing downward, perpendicular force [also known as the normal force] of the incline on the block, and the force of static friction pointing parallel to the incline opposite to the direction the block may slide).
   b) Draw a set of axes with one axis parallel to the incline and one axis perpendicular to the incline. Notice that two of the forces fall along these axes.
   c) Draw the two components of the weight along these axes. Use a scale diagram or trigonometry (sines and cosines) to find the values of these components.
   d) Because the block is at rest (not accelerating), all the forces must add up to zero. Use this fact to find the normal force and the force of static friction.



2. Imagine that you can vary the angle of the incline in the previous problem. As you increase the angle, at first, the block just stays where it is. However, at some angle $\theta$, the block begins to slide down the incline. Let the mass of the block be $m$ so you can work only with symbols. The coefficient of static friction is equal to the tangent of this angle.

$$\mu_s = \tan\ \theta$$

(Tangent is a trigonometry function that you can find on many calculators.)

a) Find the frictional force by pulling a block at a constant speed across a horizontal table. Calculate $\mu$.

b) Find the $\mu$ by tilting the table so that once started, the block can move with a constant speed. Measure the angle of the tilt, and calculate $\tan\ \theta$.

c) Compare the values in *a)* and *b)*.

## What Do You Think Now?

At the beginning of this section, you were asked the following:

- **Why do some sports require special shoes?**
- **Why would different features of a shoe be useful for different sports?**

Record your ideas about these questions now. Use the concept of sliding friction to answer the questions this time.

## Physics Essential Questions

### What does it mean?

An athlete may complain that the field is slippery. How can you describe the same situation using the terms friction, coefficient of sliding friction, forces, and the symbol $\mu$?

### How do you know?

A running shoe is pulled along the ground at a constant speed. How do you know that a frictional force was equal to the pulling force?

### Why do you believe?

| Connects with Other Physics Content | Fits with Big Ideas in Science | Meets Physics Requirements |
|---|---|---|
| Force and motion | Change and constancy | * Experimental evidence is consistent with models and theories |

* Physicists often believe in invisible forces. Friction is invisible; it happens without you seeing it. A sliding object slows down and stops. If you did not believe in friction, could Newton's first and second laws explain this motion?

### Why should you care?

Friction, or the lack of enough friction, is critical to sports. Describe two parts of your sport where friction is critical and what you are going to say about it in your voice-over.

## Reflecting on the Section and the Challenge

Many athletes seem more concerned about their shoes than other items of equipment, and for good reason. Small differences in the shoes (or skates or skis) can affect performance. Athletic shoes have become a major industry because people in all "walks" of life have discovered that athletic shoes are great just about anywhere. Now that you have studied friction, you know about a major aspect of what makes shoes function well. You are prepared to do physics commentary on athletic footwear and other effects of friction in sports when the need exists to be "sure-footed." Your sports commentary may discuss the $\mu$ of the shoe, the change in friction when a playing surface gets wet, and the need for friction when running. You may also wish to discuss the use of cleats on certain surfaces or the friction of tires on the road in stock car races. No matter which sport you choose, friction will play an important role.



## Physics to Go

1. Think of a sport and changing weather conditions that would cause an athlete to want to increase friction to have better footing. Name the sport, describe the change in conditions, and explain what the athlete might do to increase friction between the shoes and the surface of the ground.
2. Think of a sport in which athletes desire to have frictional forces as small as possible and describe what the athletes do to reduce friction.
3. If a basketball player's shoes provide an amount of friction that is "just right" when she plays on her home court, can she be sure the same shoes will provide the same amount of friction when playing on another court? What details about the other court would she need to know to answer this question?
4. Tennis is played on clay, grass and hard surfaces. Please explain why you think tennis players have or don't have different shoes for each surface.
5. A cross-country skier who weighs 600 N has chosen ski wax that provides $\mu = 0.03$. What is the minimum amount of horizontal force, perhaps from a tailwind, that would keep the skier coasting at constant speed across level snow?
6. A vehicle having a mass of 1000 kg had an accident on a wet, but level, concrete road under foggy conditions. The tires were measured to have $\mu = 0.55$ on wet concrete. The driver locked the brakes, skidded for 6 seconds, and then hit the guardrail causing a very small dent because the vehicle stopped just as it touched the guardrail. The driver claimed to be driving 65 miles per hour (29 m/s). You have been hired as an investigator to determine if the driver is telling the truth.
   a) What is the weight of the vehicle?
   b) The frictional force produces the negative acceleration (often called deceleration) that reduces the velocity of the vehicle from its initial unknown speed to zero. Find the value of the frictional force.
   c) Use the frictional force to calculate the acceleration (remember that it is a negative number).
   d) Using the acceleration and the time over which acceleration occurred, calculate the change in speed that the acceleration would produce.
   e) Use the change in speed to find the original speed of the vehicle when the brakes were applied. Write a statement of your findings, including your opinion of the driver's claim.

7. In some sports, the air or water have limiting effects on motion similar to sliding friction. Do you think that the forces of "air resistance" and "water resistance" remain constant or do they change when speeds change? Use examples from your own experience with these forms of resistance as a basis for your answer.
8. If there is a maximum frictional force between your shoe and the track, does that set a limit on how fast you can start (accelerate) in a sprint? Does that mean you cannot have more than a certain acceleration even if you have incredibly strong leg muscles? What is done to solve this problem?
9. How might an athletic shoe company use the results of your experiment to "sell" a shoe? Write copy for such an advertisement.
10. Explain why friction is important to running. Why are cleats used in football, soccer, and other sports?
11. ***Preparing for the Chapter Challenge***
    Choose a sport and describe an event in which friction with the ground or the air plays a significant role. Create a voice-over or script that uses physics to explain the action.

## Inquiring Further

**Frictional force and the weight of a player**

You found that for a given coefficient of sliding friction, the frictional force increased with the weight of the shoe. Of course, if a person is wearing the shoe, it is the weight of the person that determines the frictional force (assuming the person has only one foot on the surface). This means that the heavier the athlete, the greater the frictional force. Does this imply that the heavier a person is, the greater his or her acceleration can be?

a) Calculate the force of sliding friction for a 50-kg person using a shoe with a $\mu$ of 0.6.

b) Calculate the acceleration of the 50-kg person due to the force of sliding friction.

c) Calculate the force of sliding friction for a 90-kg person using a shoe with a $\mu$ of 0.6.

d) Calculate the acceleration of the 90-kg person due to the force of sliding friction.

e) Can the heavier person achieve a greater acceleration using the same shoes?